

**A MATHEMATICAL MODEL FOR CALCULATING THE EFFECT OF TOROIDAL GEOMETRY  
ON THE MEASURED MAGNETIC FIELD**

A RESEARCH PAPER  
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BY  
BRENDA SKOCZELAS  
DR. RANJITH WIJESINGHE – ADVISOR

BALL STATE UNIVERSITY  
MUNCIE, INDIANA

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## ABSTRACT

A mathematical model to calculate the measured magnetic field from a stimulated nerve has been presented in the past. Traditionally, electrodes have been used to measure these propagating action signals in nerves, but a less invasive technique is to use toroids. However, up until now, when using a toroidal transformer to record the nerve action currents, the thickness of the toroid has yet to be considered in the model and how it may affect the propagating compound action potential. In this paper, I will discuss the development of a new model, to which the thickness of the toroid is taken into account. These dimensions are important because the toroid represents an inhomogeneity in the extracellular medium that redistributes the extracellular current. In the past, toroids with very small diameters have been used and as they may not disrupt the action current, offer low spatial resolution. With a better understanding of the toroidal effects, we may be able to increase the accuracy and dependency of such measured magnetic signals. The final goal will be to compare the theoretical model to experimentally gathered data.

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# **CHAPTER 1**

## **Introduction and Overview**

### **1.1 Introduction**

Our body is an intricate circuit system. It is composed of hundreds of axonal pathways that have the ability to carry information by means of electrical pulses, which are known as action potentials (APs) or nerve impulses. The AP allows excited cells to carry a signal over a distance. The experienced sensations are not a reproduction of the actual stimuli, but symbols that can be used to inform us of the surrounding physical environment [14]. It is imperative that we understand how this quantitative information is being transported to the central nervous system. The first explanation and studies of axonal electrical activity arose from experiments performed by English physiologists A. L. Hodgkin and A. F. Huxley in the early 1950's. The experiments were performed on a giant axon of the squid. They found that specific voltage –dependent ion channels control the flow of ions through the cell membrane. A series of five papers were published describing their findings and are still referred to today for the basic laws that govern the movement of ions during an AP [8]. They introduced new innovative experimental techniques for characterizing membrane properties, as well as, a theoretical



model that helped to form our current knowledge of axonal excitability. The Hodgkin and Huxley model was able to demonstrate the fundamental concepts of the AP. They both later went on to be awarded the 1963 Nobel Prize in Physiology and Medicine for their work.

## 1.2 Overview of Action Potential

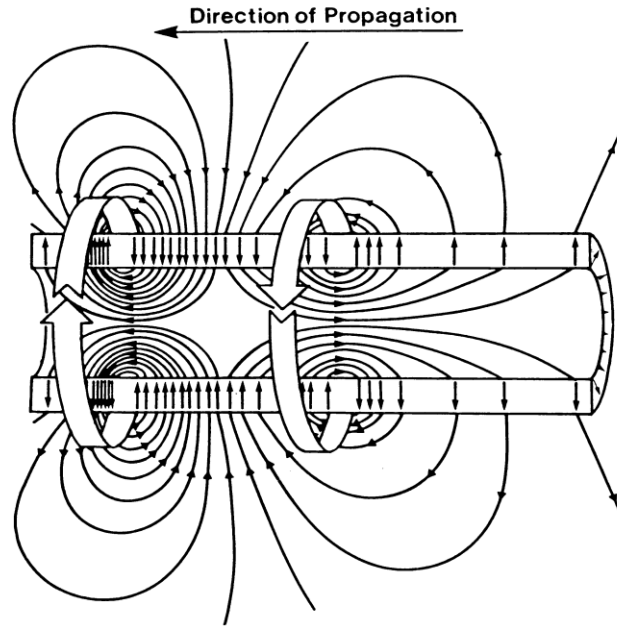
Action potentials allow for our movement and thinking. A nerve bundle contains thousands of single nerve fibers or also called nerve axons. Each nerve axon has a semi-permeable and electrically polarized limiting membrane that participates in the propagation of the energy known as the nerve impulse. The potassium-ion content inside and outside the axon is the main contributor for the difference in potential across its surface. The difference is about 65 times larger inside than outside the axon. Electrophysiologists routinely measure the changes in this electric potential in single resting and active nerve cells. In the resting membrane, the resting potential is constant and there is *no net current* crossing in or out of the cell. The action potential is the result of changing the membrane from a  $K^+$  selective condition to a  $Na^+$  selective condition [14]. Electric currents flow longitudinally in the internal and external compartments of the nerve, as well as radially through the membrane, between the inside and the outside of the cell. The nerve experiences depolarization and then repolarization that is driven by a potential change of the order of 70 mV across the membrane. The peak currents range from 5 to 10 microamps [13]. Just as a magnetic field surrounds a long, straight wire carrying a current, a small magnetic field surrounds the self-propagating action potential or wave of depolarization [7]. The field is proportional to the current flow [6]. The

strength of the external magnetic field can be estimated from Ampere's law, in which  $I$  is the net axial current enclosed by a closed path of integration  $c$

$$\oint_c B \cdot dl = \mu_o I . \quad (1-1)$$

### 1.3 Summary of Past Research in Experimentally Measured Magnetic Fields

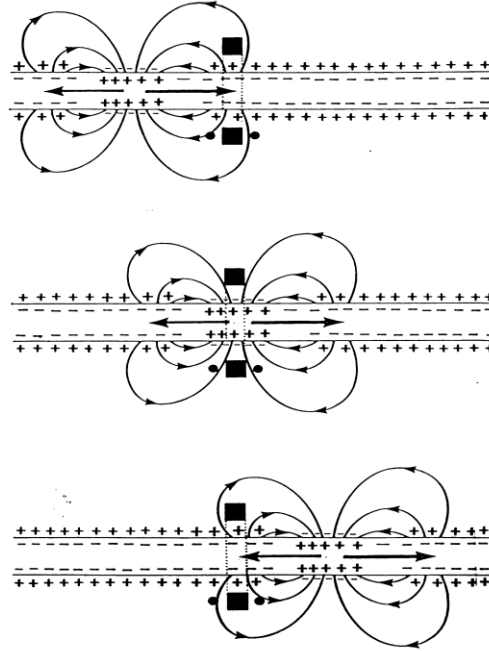
An action potential propagates along a nerve if the stimulus has a magnitude equal to the threshold value for excitation. The response travels in both directions from the point of stimulation and obeys the *all-or-none-law* [14]. There are several ways to create a stimulus. Two such techniques are magnetic and electrical stimulation. For my model, I will assume a brief electric current as an artificial stimulus. It is highly probable that the process of excitation itself is electrical. The magnitude of the AP is independent of the magnitude of the stimulus, provided that it is not smaller than the threshold. The response is characteristic of the reacting structure, i.e. properties of the nerve. A nerve action potential has the form of a moving, azimuthally symmetric solitary wave. It can be modeled as two opposing equivalent current dipoles. A small magnetic field is induced by the nerve action impulse along an excited nerve [18].



**Figure 1.1** Diagram shows the magnetic fields (thick arrows) and current lines (thin arrows) associated with a stimulated nerve [17].

The first magnetic field measurements were made by John P. Wikswo at Vanderbilt University in 1980. If the nerve is immersed in a conducting medium, the maximum magnetic field of 1 nT occurs at the nerve surface ( $r \leq 0.3$  mm) [18]. The extremely small magnetic field of the nerve bundle can be measured using either a Superconducting Quantum Interference Device (SQUID) or a specially developed low noise, room temperature amplifier along with a copper –wound toroidal pickup coil [13]. For my research project, I will be assuming the second technique, because then we have the proper equipment to later test the mathematical model. Therefore, the model will have the stimulated nerve threaded through a ferrite core toroid, both of which are immersed in a Ringer’s solution. As the action potential propagates along the nerve, the current will induce a magnetic field around the nerve, which will induce a current in the toroid by Faraday’s Law [6]. It is the net current through the toroid that is detected by the

amplifier. The net current is the sum of the intracellular and extracellular currents, as seen by the current lines shown below.



**Figure 1.2** The two thick arrows represent the intracellular current and the thin lines represent the extracellular current. The thick blocks show a cross section of the surrounding toroid around the nerve [17].

#### 1.4 Research Goals

I plan to develop a theoretical model of a single stimulated nerve axon threaded through a toroid. It will allow us to observe the induced potential at the surface of the toroid, as well as calculate both the extracellular return current and total net current through the surrounding toroid. I hope to illustrate the potential distribution outside the axon and near the toroid. The model will show the effects on the redistribution of the current lines in the conducting bath from different toroidal geometries.

## **CHAPTER 2**

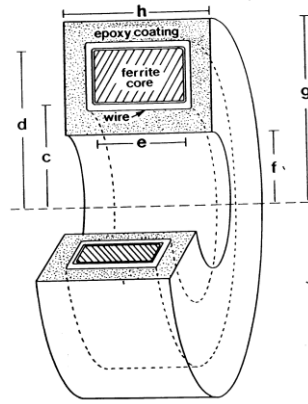
### **Mathematical Model**

#### **2.1 Volume Conductor Model**

I will follow the approach that is used often to calculate the compound action potential inside and outside a nerve bundle. My mathematical model will be based on a single fiber, volume conduction model. I assume an infinitely long cylindrical nerve immersed in a homogenous conducting medium and enclosing a nerve axon that is centered in the nerve bundle. There is also an assumption that the properties are consistent in the axial direction, since we are not considering a myelinated axon. The AP propagates with a uniform conduction velocity along the axon [12]. The conductivities interior to the axon and exterior to the nerve bundle are linear, homogeneous, quasistatic and isotropic. The electric potentials in the regions where the conductivity and properties are the same in all directions can be calculated by solving Laplace's equation [15]. As the AP propagates down the nerve, I will be analyzing the potential and current for several snapshots in time.

## 2.2 Improving on the Mathematical Models

Prior to this paper, no theoretical model has taken into consideration the effects of the surrounding toroid on the measured magnetic field. The magnetic field is directly related to the current flow inside the toroid. As the distance the toroid is placed from the nerve is increased, an increasing fraction of the external current returns within the closed path of integration, so that the magnetic field at 1 cm is a few picoteslas and decreases thereafter in proportion to the inverse cube of the distance [18]. There is an assumption that the return current has little effect on the measured magnetic field, however it has been observed in experimental data [12]. It is important that we know the current distribution both in and around the excited axon, in order to obtain information on the magnetic field. Thus far, there has been no model to illustrate the effect of the toroid on the external current distribution. The toroid represents an inhomogeneity in the external bath. The toroid is insulated from the bath by a layer of epoxy coating, which allows for no current to flow through the toroid. In order for the current to make a closed path, it must either go around the outside of the toroid or back through the center creating a return current that will cancel out the primary current in axon. If the toroid does not fit close to the axon, there can possibly be a significant change in the measured magnetic field [12]. We need to incorporate into our model, the dimensions of the toroid and epoxy coating.



**Figure 2.1 Toroid Geometry [12]**

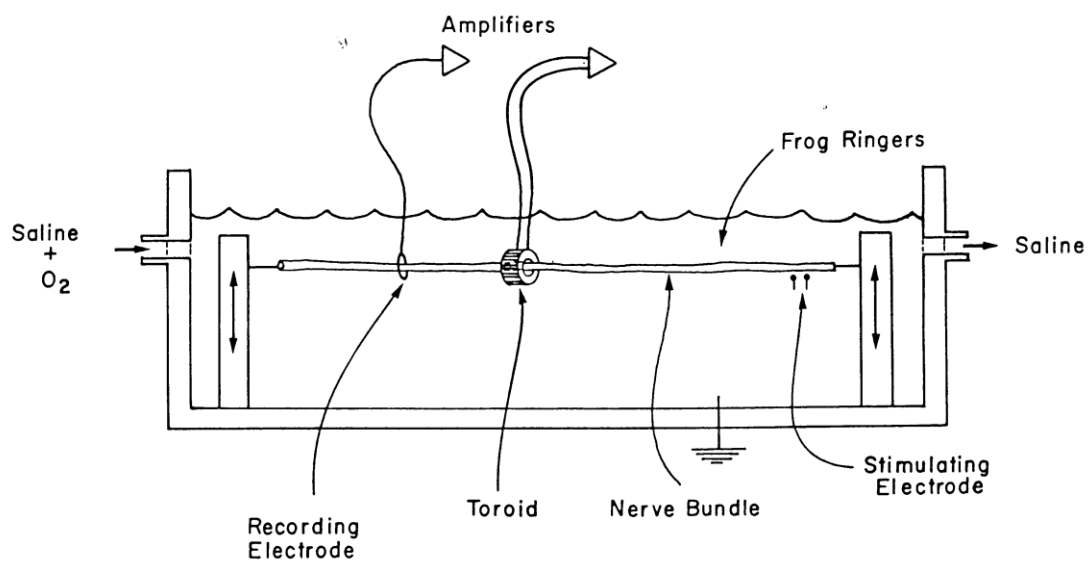
### 2.3 Model Setup

The mathematical model will be set up much the same way as the eventual experimental setup in the laboratory, in order to better test the validity of the model. I will assume a single axon of length 10 cm will be threaded through a toroid and placed at the center of a plastic container 10 cm x 5cm, which is filled with Ringer's solution. The depth of the container is not important, because we will be looking at an over head cross section for our model. There will be restraints on the extracellular current flow. No current can pass through the boundaries of the plastic container, as well as, no current can flow into or out of the toroid. This type of constraint is known as a Neumann boundary condition [9]. On the boundary surfaces we have

$$\frac{\partial V}{\partial n} = 0, \quad (2-1)$$

where  $n$  indicates the direction perpendicular to the surface. The external potential values can be derived analytically, but as we have discovered, that will be very difficult.

Therefore, we have chosen a numerical method, to which our experimental setup works well.



**Figure 2.2** A schematic representation of the experimental setup.



## CHAPTER 3

### Numerical Method

#### 3.1 Relaxation Method

In order to determine the extracellular potential values on the surface of the toroid and throughout the conducting medium, I will use the special technique of relaxation. The relaxation method is a numerical approach to solving Laplace's equation for boundary-value problems [3]. I will assume our system is that of conductors and look to solve for the electric potential at specific snapshots in time. The general equation,

$$\nabla^2 \Phi = 0, \quad (3-1)$$

can be interpreted as a differential statement of the fact that the solution at a specific point is just the average of the solutions over a surface of *any* shape surrounding the point of interest [3]. We must replace the differential equation with the finite divided differences that define a derivative [3]. Let us start with Laplace's equation for an azimuthally cylindrical coordinate system,  $\Phi = \Phi(\rho, z)$ :

$$\nabla^2 \Phi = \frac{\partial^2}{\partial \rho^2} \Phi(\rho, z) + \frac{1}{\rho} \frac{\partial \Phi}{\partial \rho}(\rho, z) + \frac{\partial^2 \Phi}{\partial z^2}(\rho, z) = 0. \quad (3-2)$$

At the point  $(\rho_0, z_0)$ , the equivalent finite difference expression is

$$\begin{aligned} & \frac{\Phi(\rho_0 + \Delta\rho, z_0) + \Phi(\rho_0 - \Delta\rho, z_0) - 2\Phi(\rho_0, z_0)}{(\Delta\rho)^2} + \frac{1}{\rho_0} \frac{\Phi(\rho_0 + \Delta\rho, z_0) - \Phi(\rho_0 - \Delta\rho, z_0)}{2\Delta\rho} \\ & + \frac{\Phi(\rho_0, z_0 + \Delta z) + \Phi(\rho_0, z_0 - \Delta z) - 2\Phi(\rho_0, z_0)}{(\Delta z)^2} = 0, \end{aligned} \quad (3-3)$$

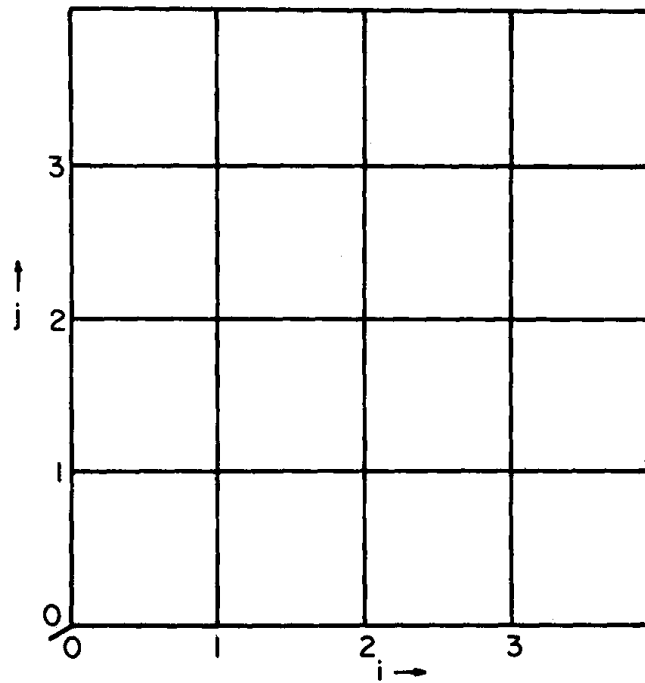
where  $\Delta\rho$  and  $\Delta z$  are the associated step sizes in the  $\rho$  and  $z$  directions. The above equation can be rearranged [3] as

$$\Phi(\rho_0, z_0) = \frac{1}{2[1 + (\frac{\Delta\rho}{\Delta z})^2]} \left[ \frac{\Phi(\rho_0 + \Delta\rho, z_0) \times \left(1 + \frac{\Delta\rho}{2\rho_0}\right) + \Phi(\rho_0 - \Delta\rho, z_0) \left(1 - \frac{\Delta\rho}{2\rho_0}\right)}{1 + (\frac{\Delta\rho}{\Delta z})^2} + \left(\frac{\Delta\rho}{\Delta z}\right)^2 [\Phi(\rho_0, z_0 + \Delta z) + \Phi(\rho_0, z_0 - \Delta z)] \right]. \quad (3-4)$$

You may notice a singularity for  $\rho_0 = 0$ , but this will not create a problem for me as I will not begin to relax the potential until  $\rho = 0.2$  mm. The mesh step size for  $\Delta\rho$  and  $\Delta z$  has been chosen to be 0.2 mm. The single axon lies along the line  $\rho = 0$  mm and has a radius of 0.2 mm.

### 3.2 Relaxation Steps

First, the continuous region outside the nerve must be replaced with a grid network [3].



**Figure 3.1 Two – Dimensional  $z$ - $\rho$  grid with  $\Phi_{ij}$ 's at free points after  $l$  iterations. The grid spacing is the same in  $z$  and  $\rho$  directions.  $z = i\Delta$ ,  $\rho = j\Delta$ , where  $i$  and  $j$  are integers [3]**

For convenience, we have set up the grid so that all boundary surfaces lie along grid lines and there is equal spacing for the  $\rho$  and  $z$  values. The two boundary surfaces are the toroid and the plastic container. The potential values on the surface of the axon are calculated using an already developed program in FORTRAN. The only change I needed to make to the values was to convert them from the time domain into the space domain. Knowing the basic distance equation, I simply multiplied time by a conduction velocity of 10 m/s to obtain  $z$  values in units of millimeters. The conduction velocity was chosen from the literature and can change depending on the characteristics of the chosen axon. The rest of the unknown extracellular potential values, “free points,” were initially approximated using the equation

$$\Phi(\rho, z) = f(z)/(1 + \rho), \quad (3-5)$$

where  $f(z)$  is the surface potential at  $z$ . Rather than choosing all the initial free points to be zero, I chose to use the above equation in order to speed up the convergence process. The relaxation process replaces the potential at a free point with the average of the neighboring points according to the above  $\Phi(\rho_0, z_0)$  equation. Once the procedure is applied to all free points, one iteration is complete. We continue until a reasonable convergence has been met [3].

### 3.3 Successive Over-Relaxation

Successive over-relaxation (SOR) is an extension of the relaxation technique introduced above. It has been found to possibly accelerate the convergence of electrostatic problems involving Laplace's equation and thus increasing efficiency of the computer programs [1,4]. A relaxation factor  $\omega$  needs to be chosen, such that  $1 < \omega < 2$ . Let  $\Phi_{ij}^l$  represent the potential value at the point (i,j) from the  $l$ th iteration using the above relaxation expression. The "over-relaxed" potential value  $\Phi_{ij}^{SOR}$  is found by the equation [3]

$$\Phi_{ij}^{SOR} = (1 - \omega) * \Phi_{ij}^{l-1} + \omega * \Phi_{ij}^l. \quad (3-6)$$

$\Phi_{ij}^l$  of the grid is then replaced by  $\Phi_{ij}^{SOR}$ .

### **3.4 Methodology**

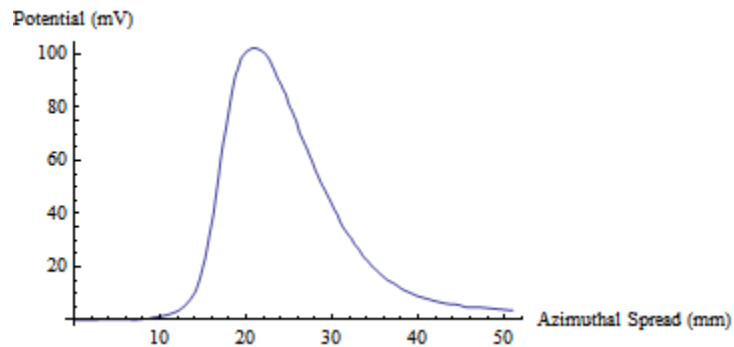
Our numerical calculations for the solution to Laplace's equation will be done using the program Mathematica 7.0. It was first released in 1988 and was considered a major advance in the field of computing. It was created by Stephen Wolfram who began Wolfram Research, Inc. Mathematica is the world's only fully integrated environment and one of the largest single application programs developed. I will be working with the latest version to date [19].

## CHAPTER 4

### Data Simulation

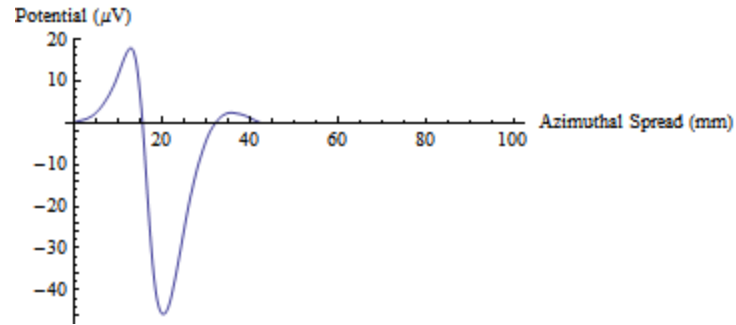
#### 4.1 Simulation of Intracellular and Extracellular Electric Potential

A nerve axon 10 cm in length lies at the center of a nerve bundle of radius 0.2 mm. The action potential propagates along the nerve at a constant velocity of 10 m/s. The AP inside the axon is shown below.



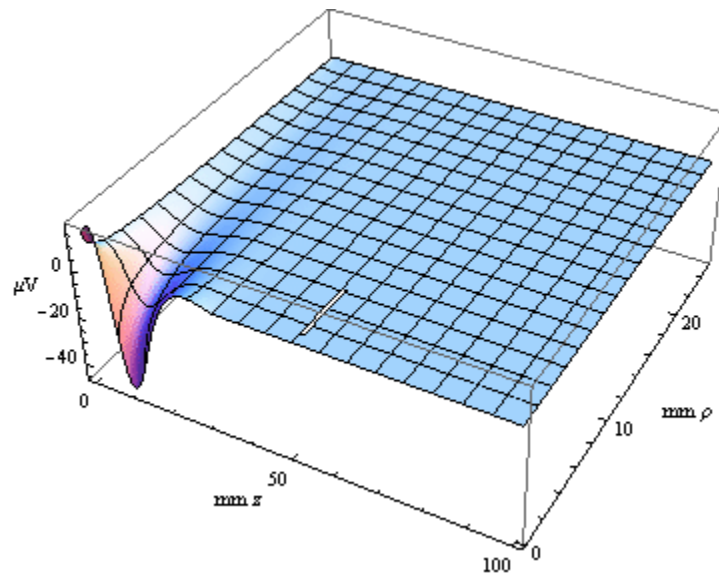
**Figure 4.1** Action potential inside the axon

The potential at the surface of the nerve is found by solving the Laplace equation outside of the nerve bundle and is shown below.

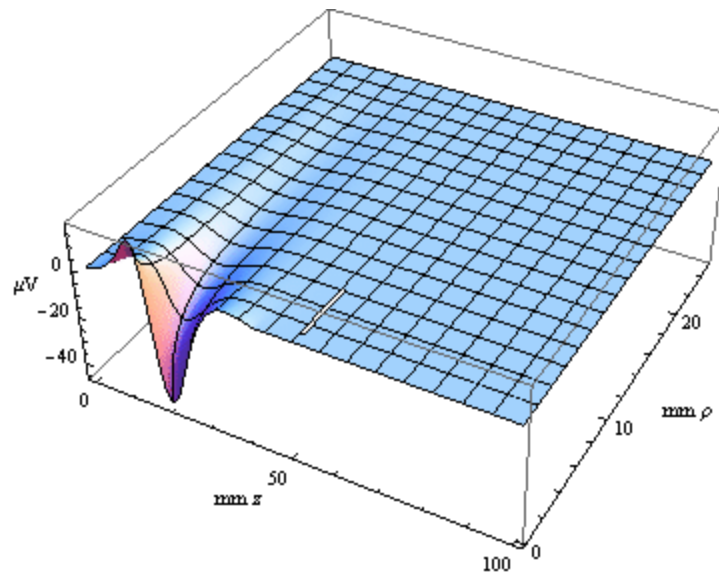


**Figure 4.2 Electric potential at surface of nerve**

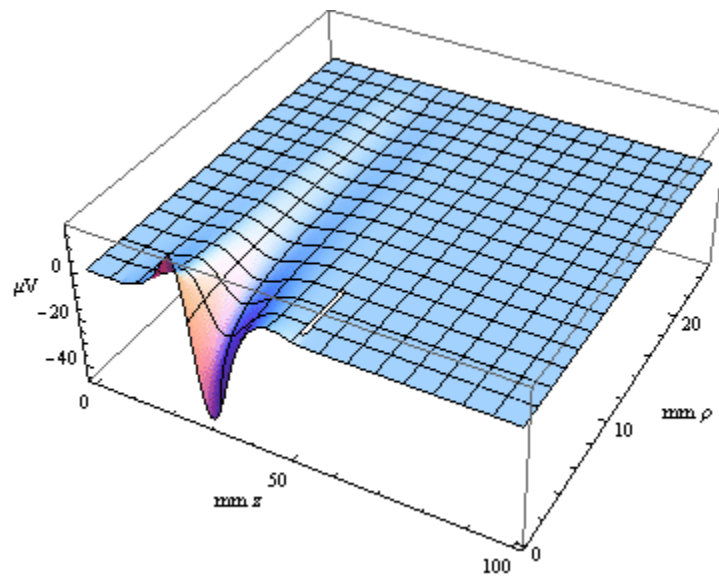
I will assume a toroid of thickness 1 mm with a difference of 4 mm between the inner radius  $f$  and outer radius  $g$  (refer back to Figure 2.1). The toroid will be positioned halfway along the nerve at  $z = 50$  mm. I will call the distance between the center of the axon and the inner surface of the toroid, the toroid radius ( $r$ ). For the first example,  $r = 1$  mm. Shown below are snapshots of the extracellular potential (microVolts) after 100 iterations as the AP propagates along the nerve.



**Figure 4.3 Extracellular potential for AP 1 cm along nerve**

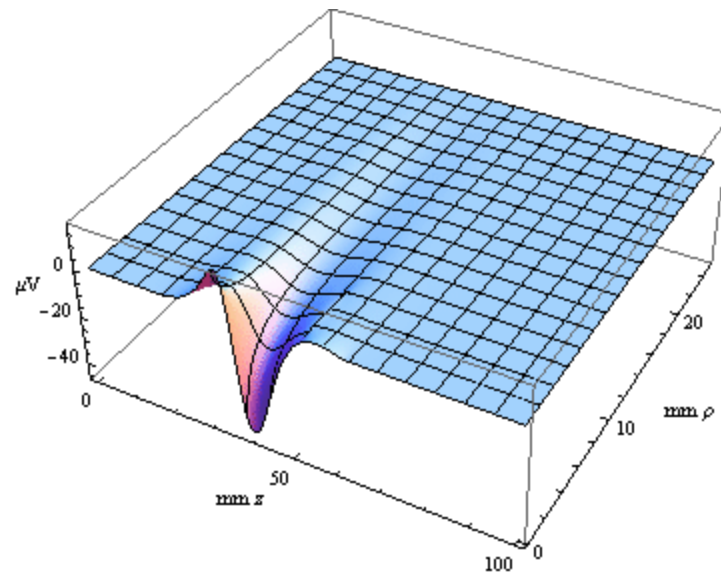


**Figure 4.4** Extracellular potential for AP 2 cm along nerve

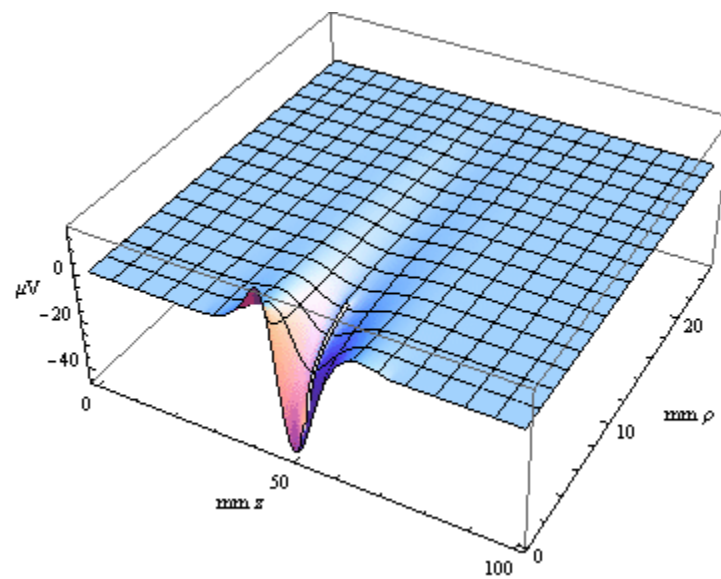


**Figure 4.5** Extracellular potential for AP 3 cm along nerve

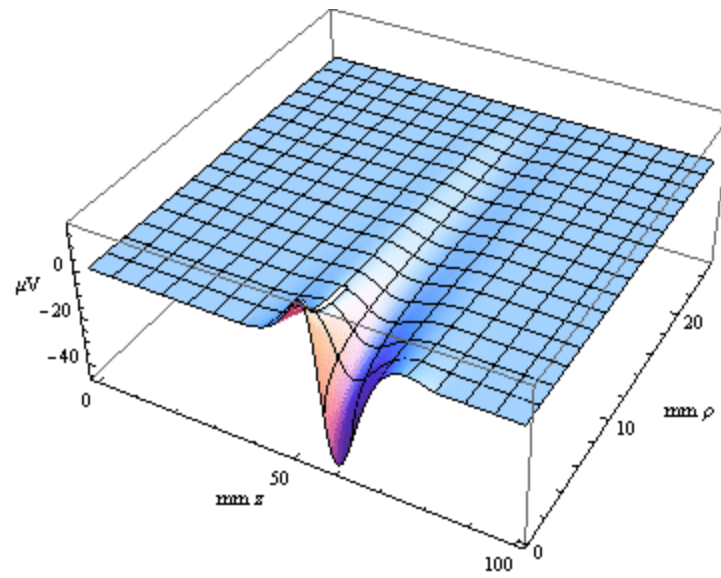




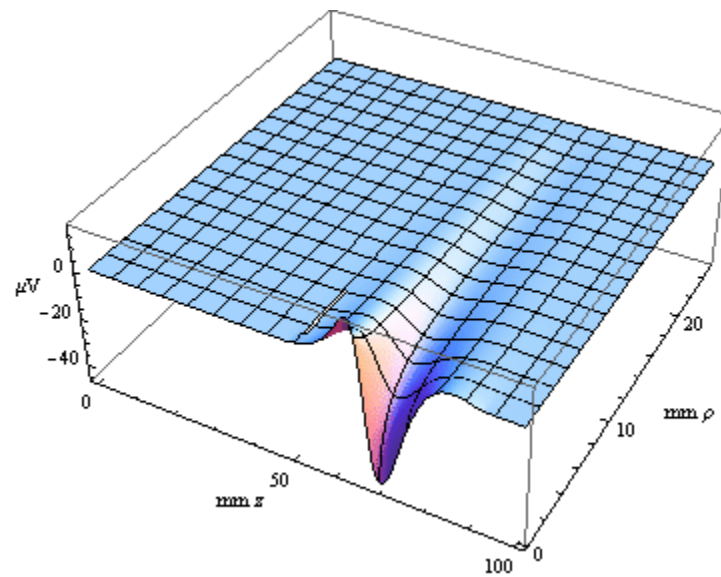
**Figure 4.6** Extracellular potential for AP 4 cm along nerve



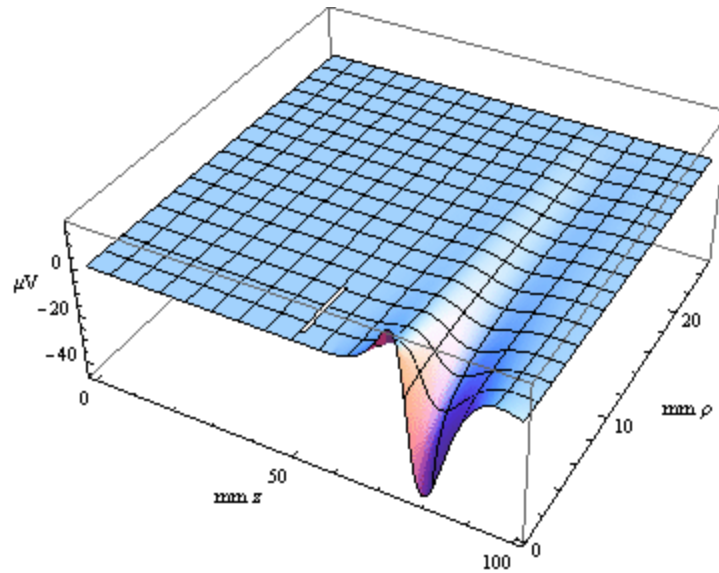
**Figure 4.7** Extracellular potential for AP 5 cm along nerve



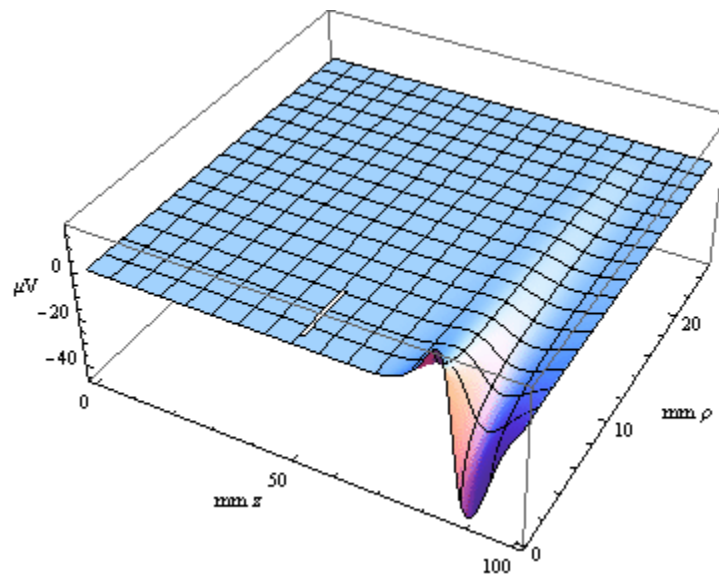
**Figure 4.8** Extracellular potential for AP 6 cm along nerve



**Figure 4.9** Extracellular potential for AP 7 cm along nerve



**Figure 4.10 Extracellular potential for AP 8 cm along nerve**



**Figure 4.11 Extracellular potential for AP 9 cm along nerve**

The traveling pulse through the toroid, causes a charge buildup on the surface of the toroid. This will give rise to an induced potential on the epoxy coating.

## 4.2 Extracellular and Intracellular Current

The extracellular potential values will be used to determine the extracellular current values. Ohm's law for the external current outside the nerve,  $I_e$ , is given by the equation

$$I_e = dV / R, \quad (4-1)$$

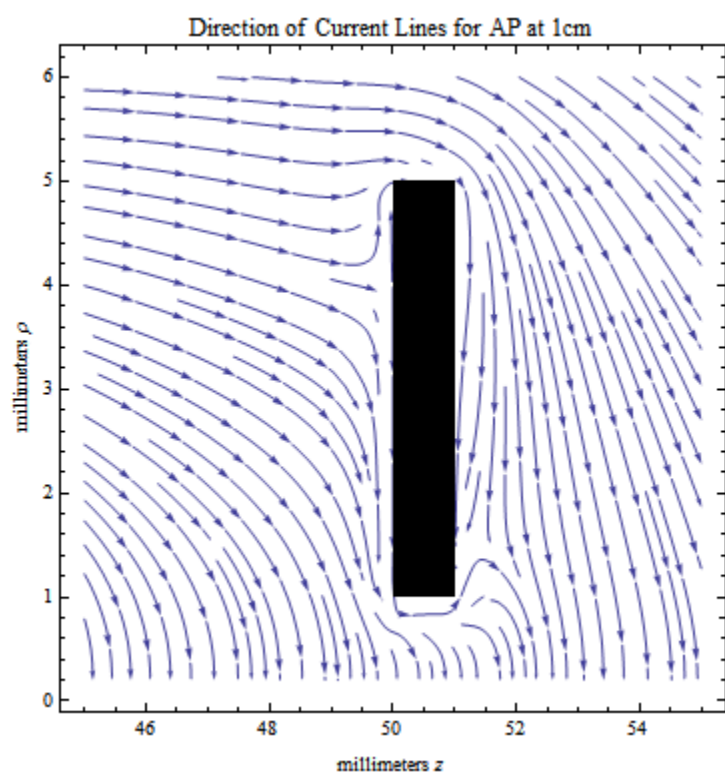
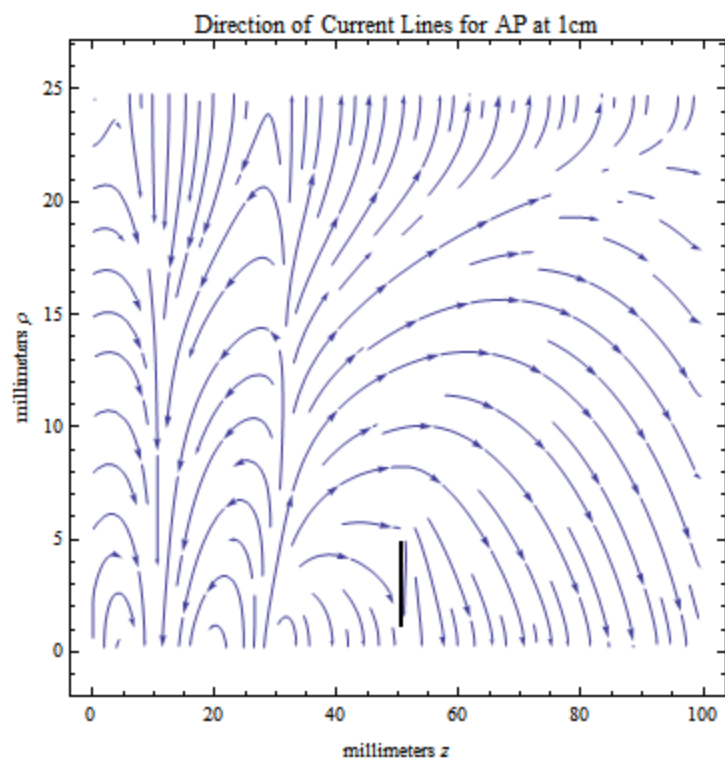
where  $R$  is the extracellular resistance per unit length and  $dV$  is the change in external potential in microVolts.  $R$  can be found by the equation

$$R = \frac{1}{\sigma_e * \Delta z}, \quad (4-2)$$

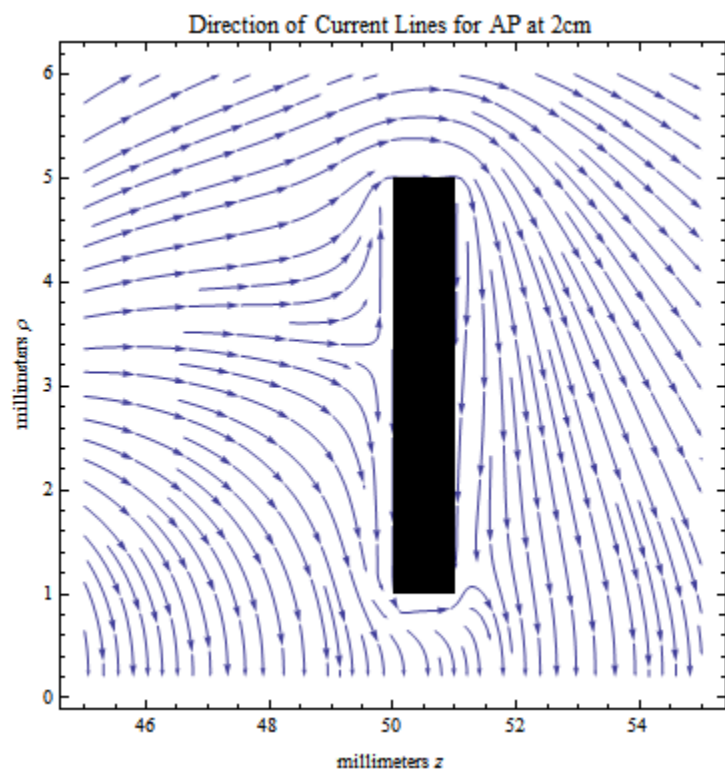
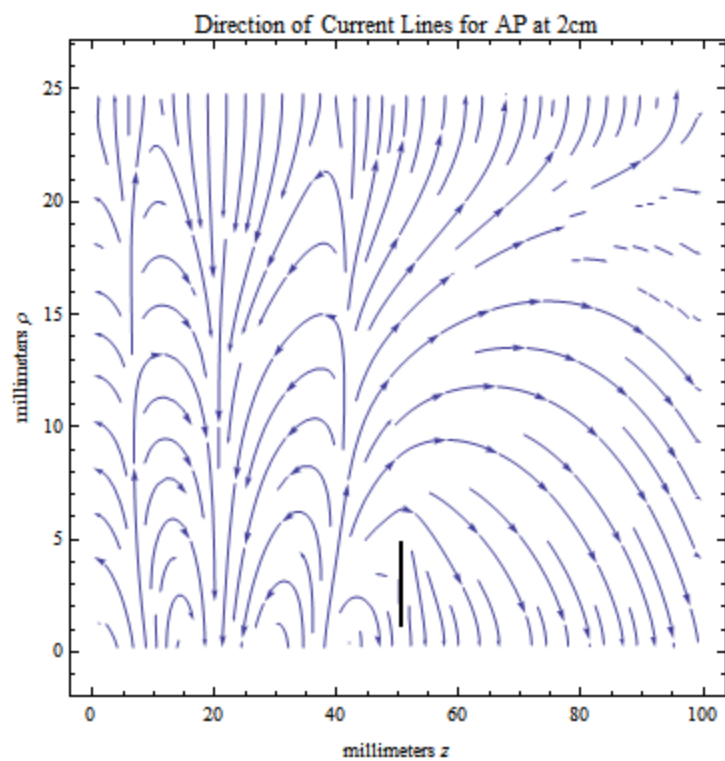
where  $\sigma_e$  is the extracellular conductivity. Ohm's law for the internal current,  $I_i$ , inside the axon is given by the equation

$$I_i = -dV/(r * \Delta z), \quad (4-3)$$

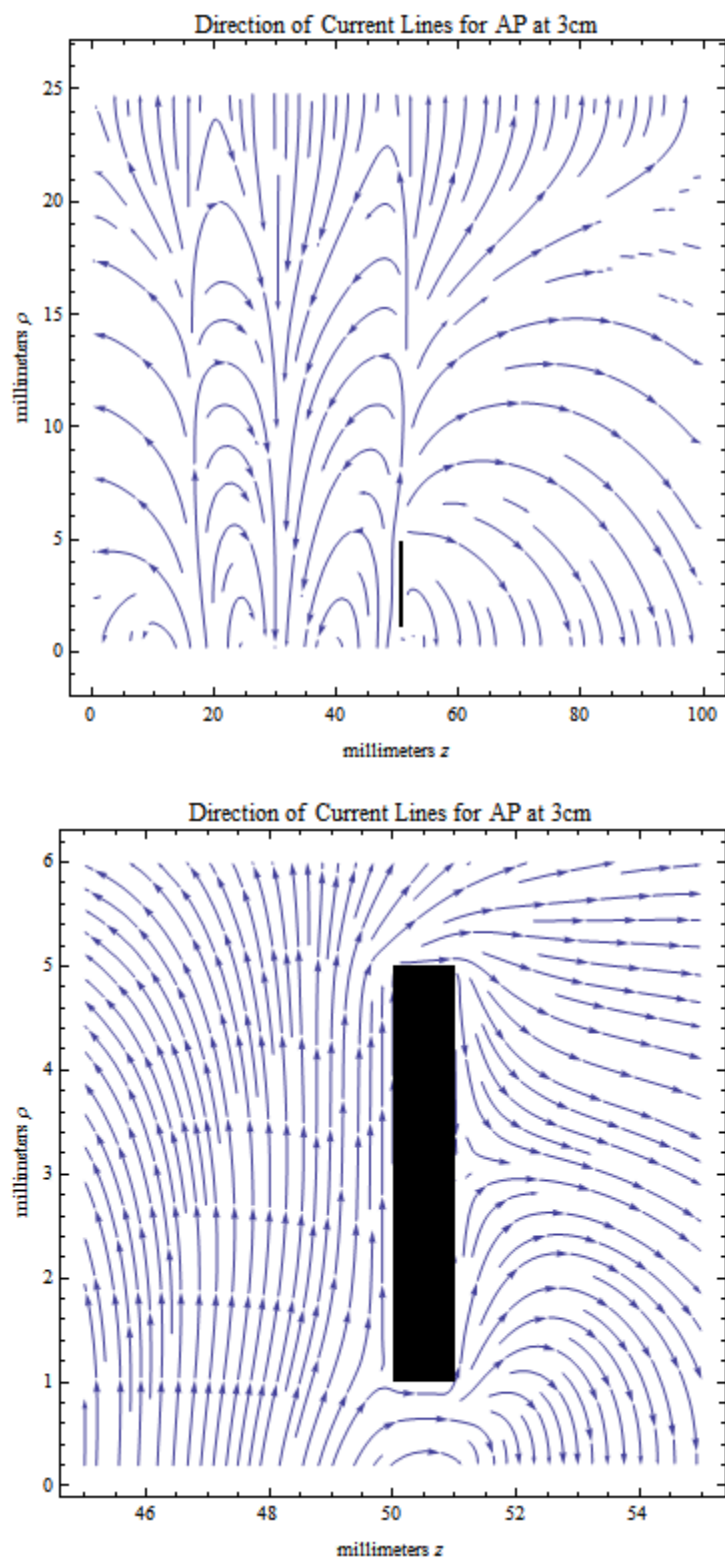
where  $r$  is the axial resistance per unit length and  $V$  is the interior potential of the axon (milliVolts). The axial resistance,  $r$ , can be found the same way as the extracellular resistance above, but with replacing the equation with the intracellular conductivity,  $\sigma_i$ . From the literature [15], we set  $\sigma_e = 1.20 \Omega^{-1}m^{-1}$  and  $\sigma_i = 0.88 \Omega^{-1}m^{-1}$ . Shown below are the extracellular current lines as the AP propagates along the nerve. For each snapshot along the axon, I show a plot for the entire axon followed by a plot zoomed in near the toroid. The black rectangle represents the toroidal boundary. I am most interested with the happenings near the toroid, because that is where the return current is seen.



**Figure 4.12** Current plots for  $r = 1$  mm



**Figure 4.13** Current plots for  $r = 1$  mm



**Figure 4.14** Current plots for  $r = 1$  mm

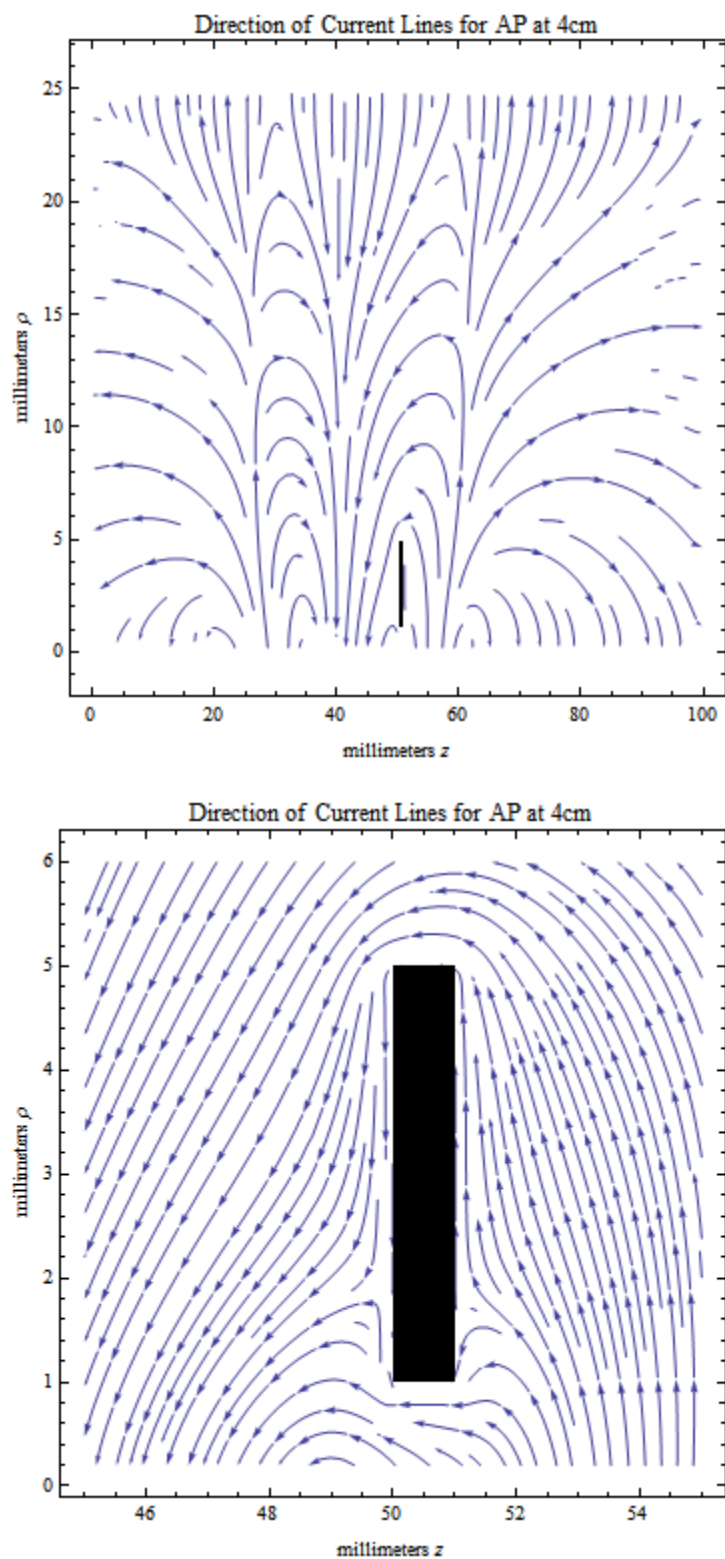
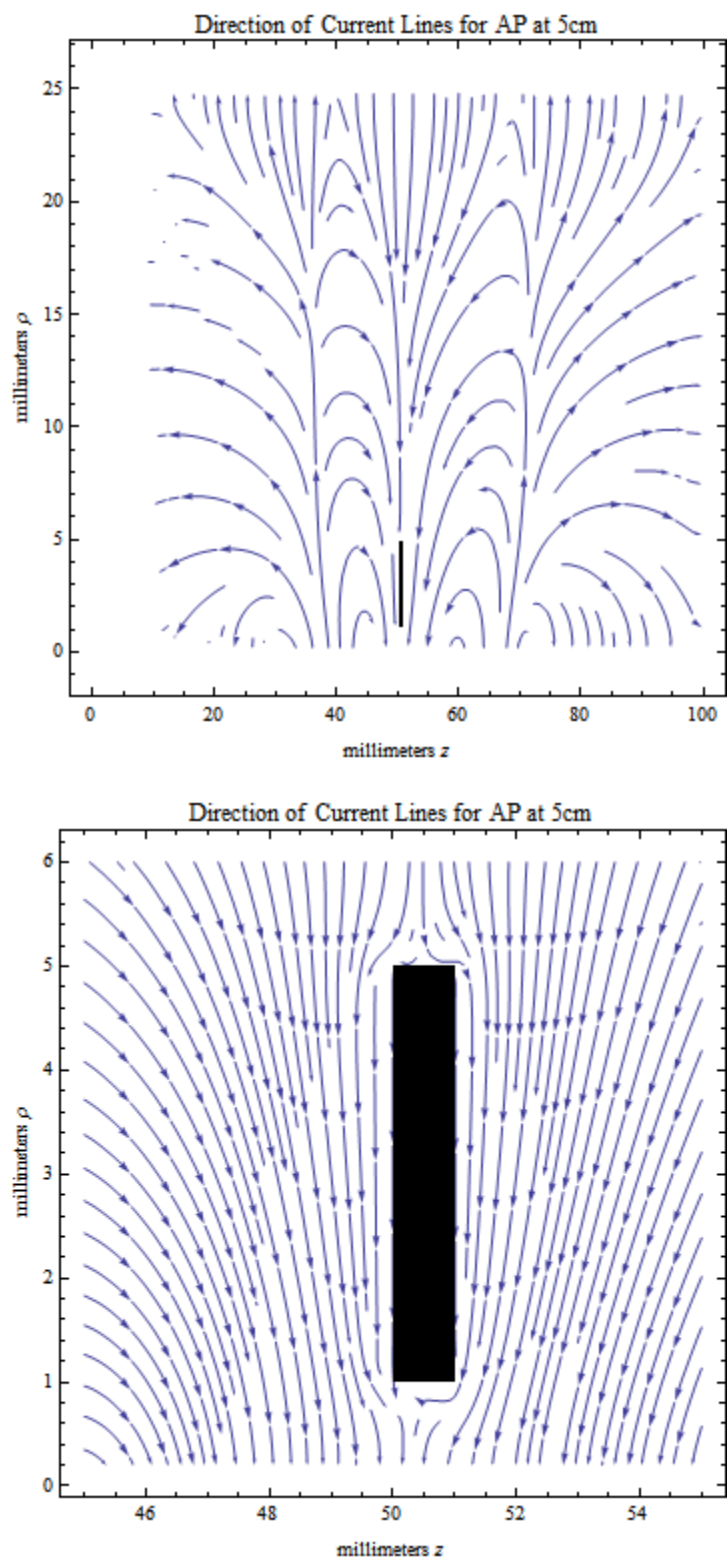
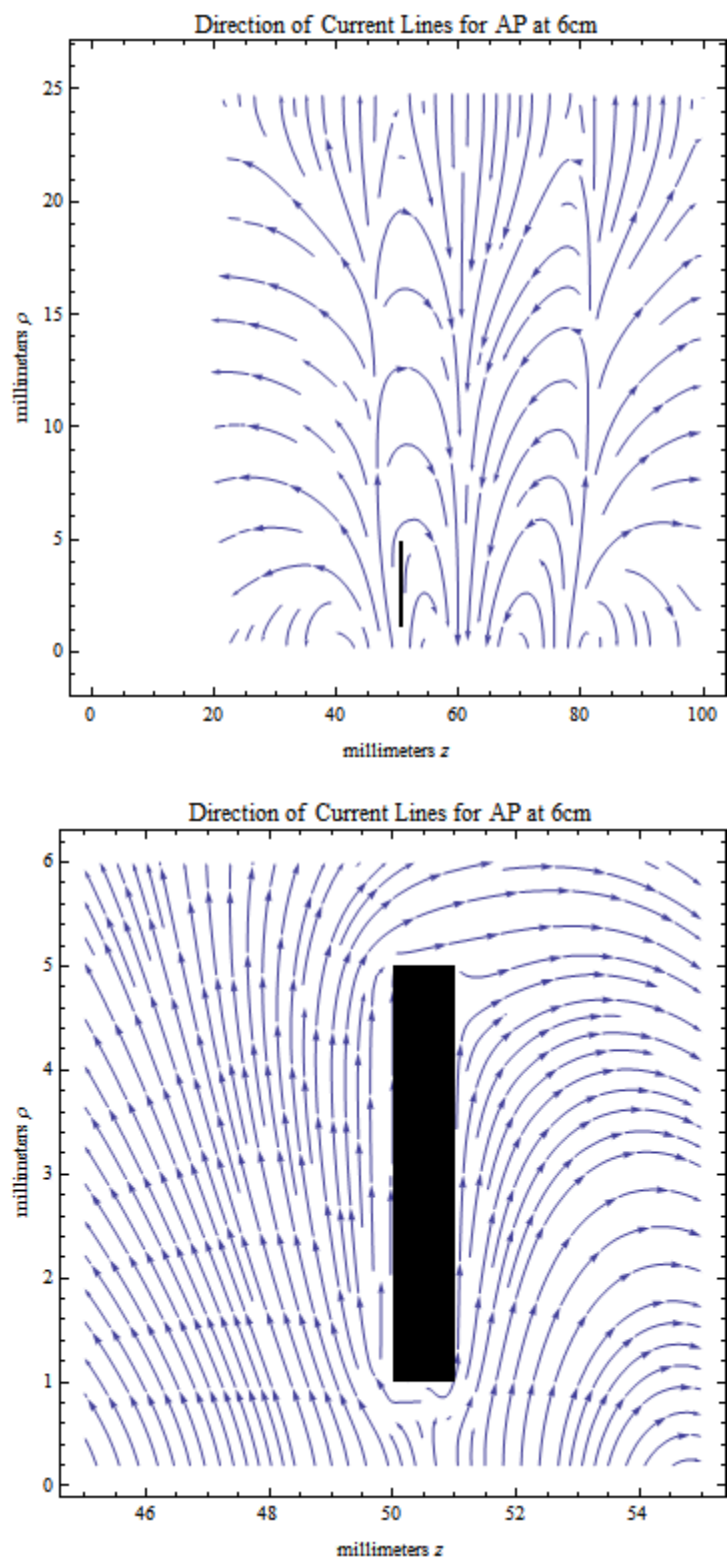


Figure 4.15 Current plots for  $r = 1$  mm

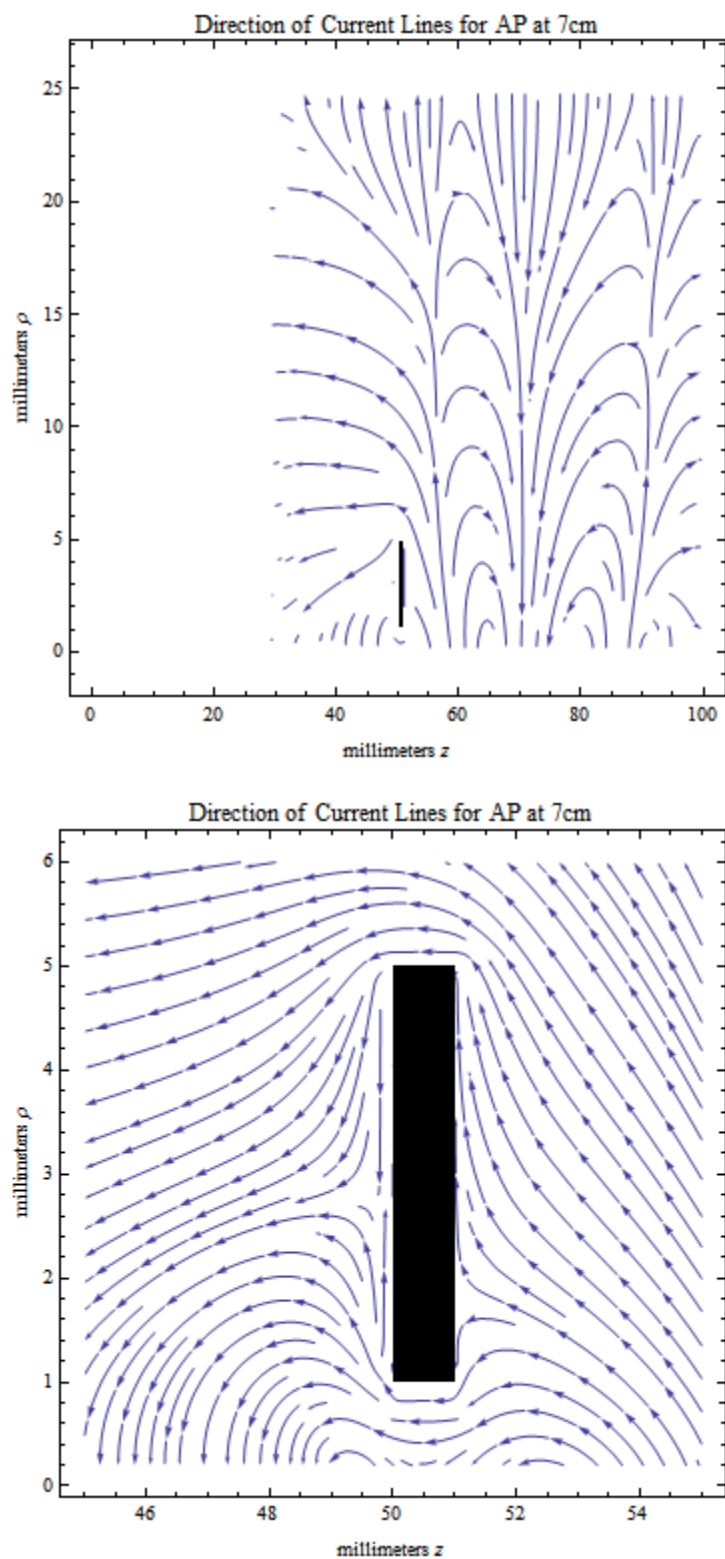




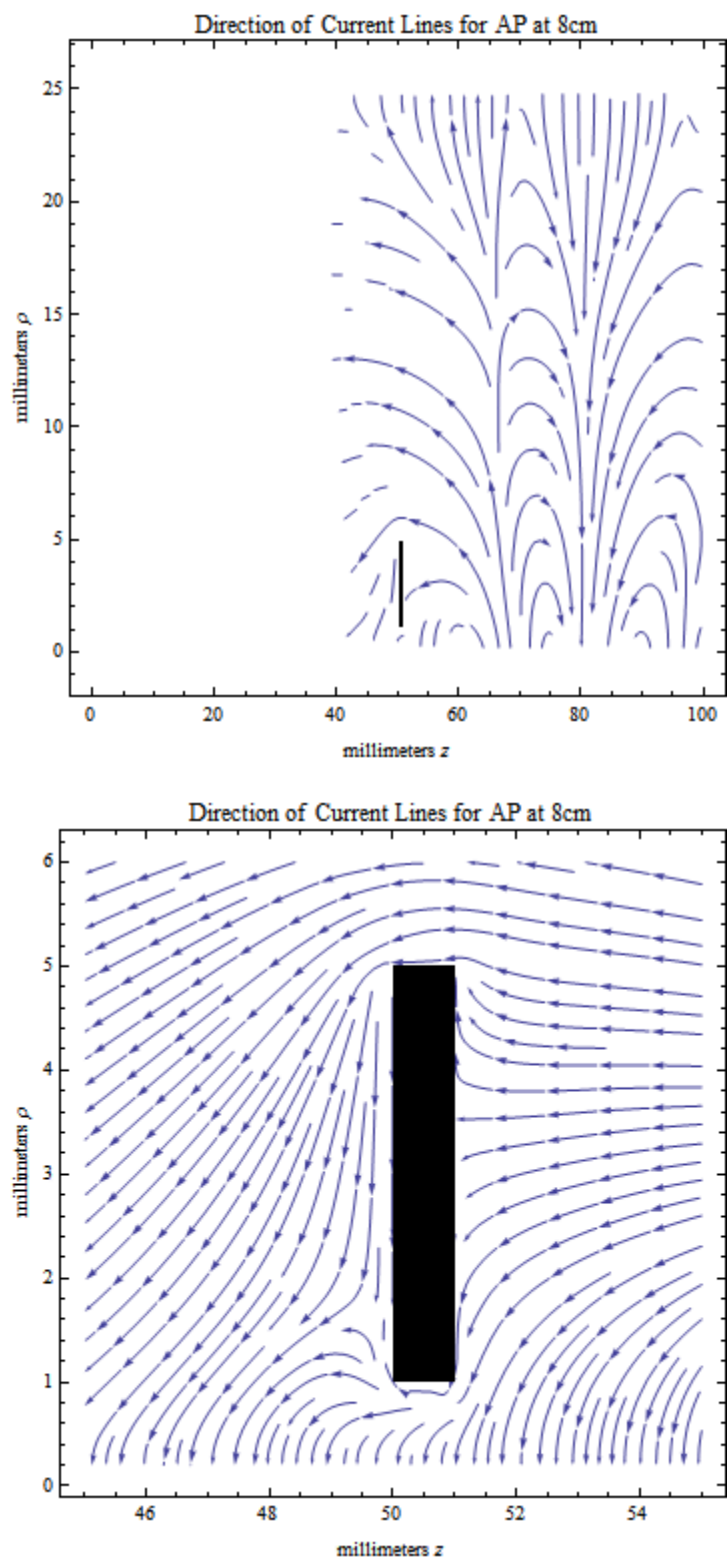
**Figure 4.16** Current plots for  $r = 1$  mm



**Figure 4.17** Current plots for  $r = 1$  mm



**Figure 4.18** Current plots for  $r = 1$  mm



**Figure 4.19** Current plots for  $r = 1$  mm

### 4.3 Extracellular Return Current

In order to calculate the total return current between the surface of the nerve and the toroid, we need only to look at the total current entering from one side of the toroid. The amount of current entering the space between the toroid and the nerve, must equal the amount of current leaving from the other side. This idea is based on the continuity principle. Using the extracellular current values that lie underneath the toroid at  $z = 50$  mm and the fact that the nerve – toroid system is azimuthally symmetric, I was able to estimate the total return current using the cross-sectional area between the toroid and the nerve.

### 4.4 Net Current

The current in and around a nerve always flows in closed loops as seen in Figure 1.2. Therefore, the intracellular and extracellular current must flow in opposite directions. I must find the intracellular current (IC) at  $z = 50$  mm and add it to the total return current (TRC) at  $z = 50$  mm, in order to calculate the net current (NC) through the toroid. It is the net current that will be picked up by the amplifier during the experiment and used to calculate the magnetic field surrounding the axon.

	<b>r = 1 mm</b>		
<b>AP (cm)</b>	<b>RC (microA)</b>	<b>IC (microA)</b>	<b>NC(microA)</b>
<b>1</b>	4.08838E-09	0	4.08838E-09
<b>2</b>	4.47848E-07	-3.18468	-3.184679552
<b>3</b>	5.15942E-05	-0.172435	-0.172383406
<b>4</b>	-0.000297349	1.0572	1.056902651
<b>5</b>	-0.000507491	0.12188	0.121372509
<b>6</b>	-0.000524863	0.167347	0.166822137
<b>7</b>	-4.80598E-05	0.010531	0.01048294
<b>8</b>	-5.52099E-08	0	-5.52099E-08

**Table 4.1 Current values for toroid radius = 1 mm**

I have also calculated the current values for four other toroid radii. The rest of the parameters used above have all stayed the same, in order to be able to best compare my results. The return current, intracellular current, and net current values for different radii are shown in the tables below. Having changed the radii values only slightly, the plots of the current lines for each individual radius all look very similar to that of 1 mm (see Figures 4.12 – 4.19). Therefore, I have not included them all. I have, however, decided to show the plot of the current lines near the toroid for  $r = 2$  mm. This shows the direction of current for the greatest distance calculated between the toroid and axon. The reader is given a nice illustration of how the extracellular current is behaving around the toroid.

	<b>r = 0.4 mm</b>		
<b>AP (cm)</b>	<b>RC (microA)</b>	<b>IC (microA)</b>	<b>NC(microA)</b>
<b>1</b>	0	0	0
<b>2</b>	0	-3.18468	-3.18468
<b>3</b>	8.82326E-06	-0.172435	-0.172426177
<b>4</b>	-3.24195E-05	1.0572	1.057167581
<b>5</b>	-4.08819E-05	0.12188	0.121839118
<b>6</b>	-5.07436E-05	0.167347	0.167296256
<b>7</b>	-4.55249E-06	0.010531	0.010526448
<b>8</b>	0	0	0

**Table 4.2 Current values for toroid radius = 0.4 mm**

	<b>r = 0.6 mm</b>		
<b>AP (cm)</b>	<b>RC (microA)</b>	<b>IC (microA)</b>	<b>NC(microA)</b>
<b>1</b>	6.47487E-10	0	6.47487E-10
<b>2</b>	7.4076E-08	-3.18468	-3.184679926
<b>3</b>	2.19539E-05	-0.172435	-0.172413046
<b>4</b>	-9.89712E-05	1.0572	1.057101029
<b>5</b>	-7.43774E-05	0.12188	0.121805623
<b>6</b>	-0.000189217	0.167347	0.167157783
<b>7</b>	-1.44133E-05	0.010531	0.010516587
<b>8</b>	-9.87178E-10	0	-9.87178E-10

**Table 4.3 Current values for toroid radius = 0.6 mm**

	<b>r = 0.8 mm</b>		
<b>AP (cm)</b>	<b>RC (microA)</b>	<b>IC (microA)</b>	<b>NC(microA)</b>
<b>1</b>	2.1115E-09	0	2.1115E-09
<b>2</b>	2.36954E-07	-3.18468	-3.184679763
<b>3</b>	3.65869E-05	-0.172435	-0.172398413
<b>4</b>	-0.000191994	1.0572	1.057008006
<b>5</b>	-0.000312543	0.12188	0.121567457
<b>6</b>	-0.000373458	0.167347	0.166973542
<b>7</b>	-2.94288E-05	0.010531	0.010501571
<b>8</b>	-1.48987E-08	0	-1.48987E-08

**Table 4.4 Current values for toroid radius = 0.8 mm**

	<b>r = 2 mm</b>		
<b>AP (cm)</b>	<b>RC (microA)</b>	<b>IC (microA)</b>	<b>NC(microA)</b>
<b>1</b>	2.15352E-08	0	2.15352E-08
<b>2</b>	2.07097E-06	-3.18468	-3.184677929
<b>3</b>	0.000122417	-0.172435	-0.172312583
<b>4</b>	-0.000904609	1.0572	1.056295391
<b>5</b>	-0.000745878	0.12188	0.121134122
<b>6</b>	-0.000713817	0.167347	0.166633183
<b>7</b>	-0.000172527	0.010531	0.010358473
<b>8</b>	-6.9044E-07	0	-6.9044E-07

**Table 4.5 Current values for toroid radius = 2 mm**

Shown below in Figures 4.20 – 4.27 are the current plots for  $r = 2$  mm.

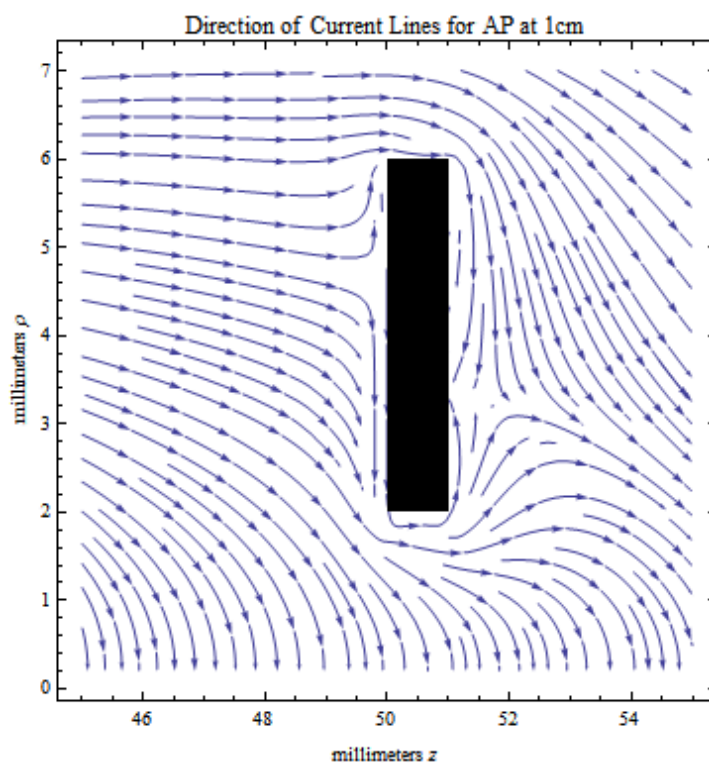


Figure 4.20 Current plot for  $r = 2$  mm

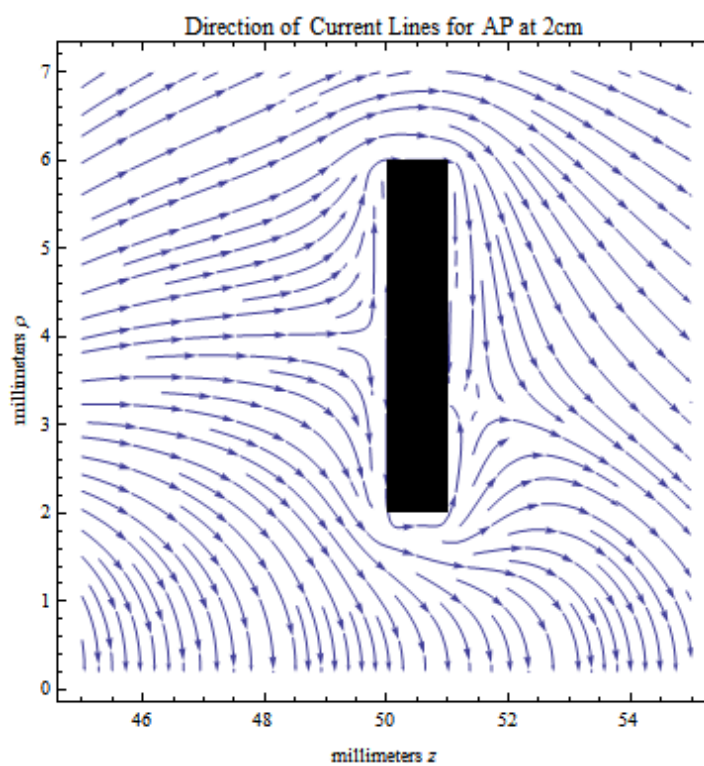


Figure 4.21 Current plot for  $r = 2$  mm



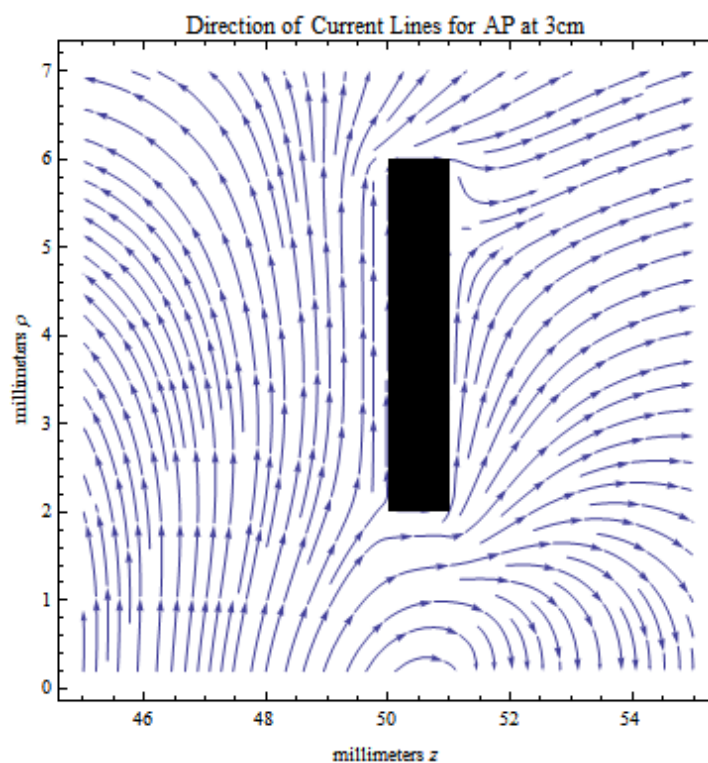


Figure 4.22 Current plot for  $r = 2$  mm

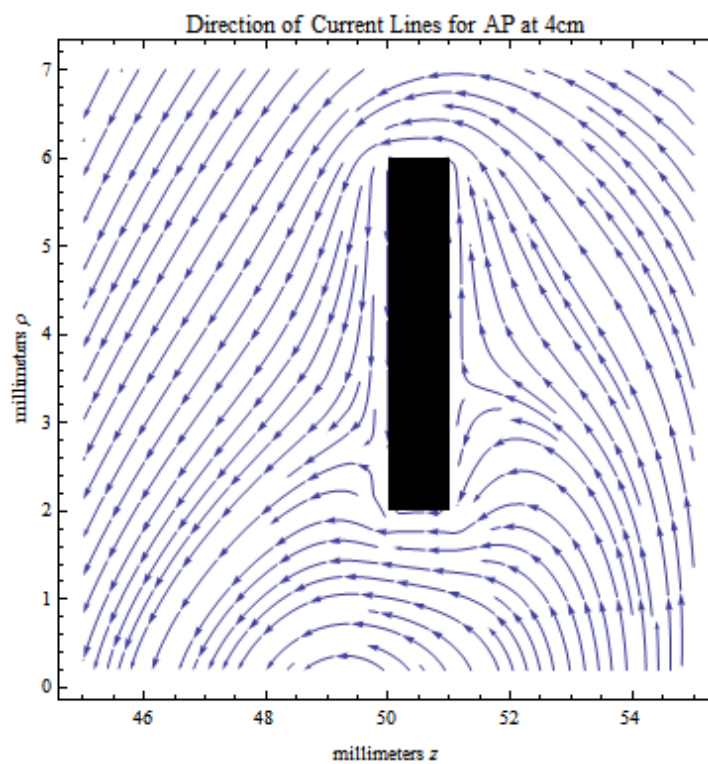


Figure 4.23 Current plot for  $r = 2$  mm

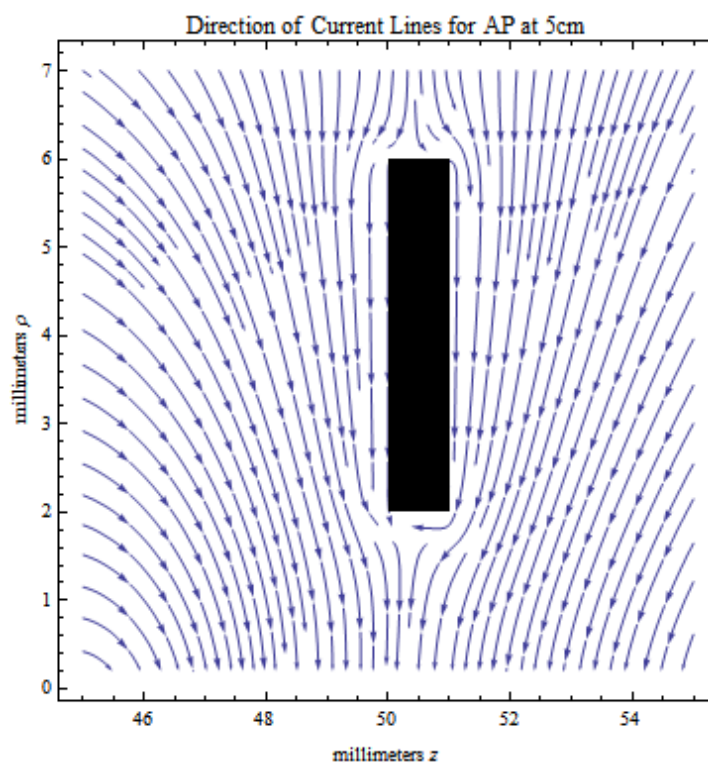


Figure 4.24 Current plot for  $r = 2$  mm

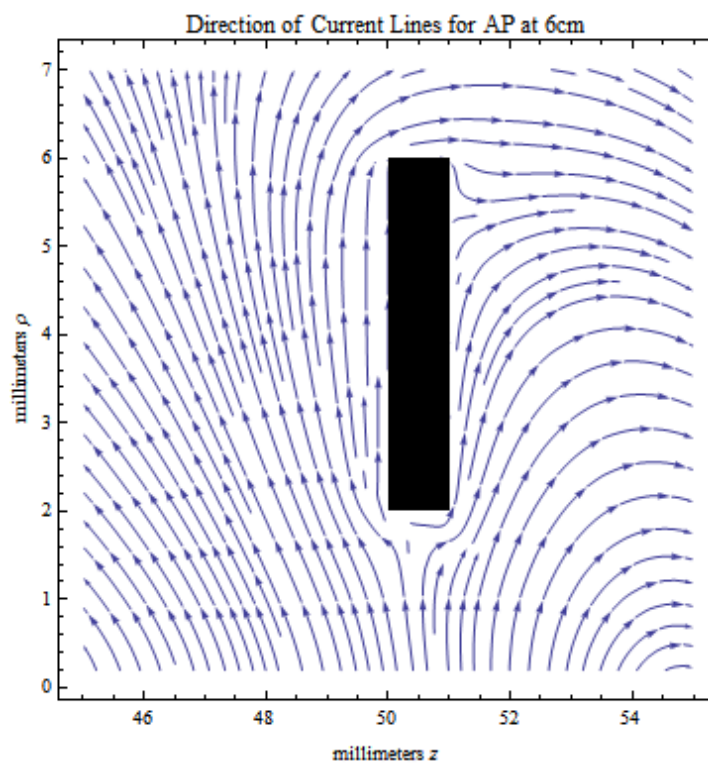


Figure 4.25 Current plot for  $r = 2$  mm

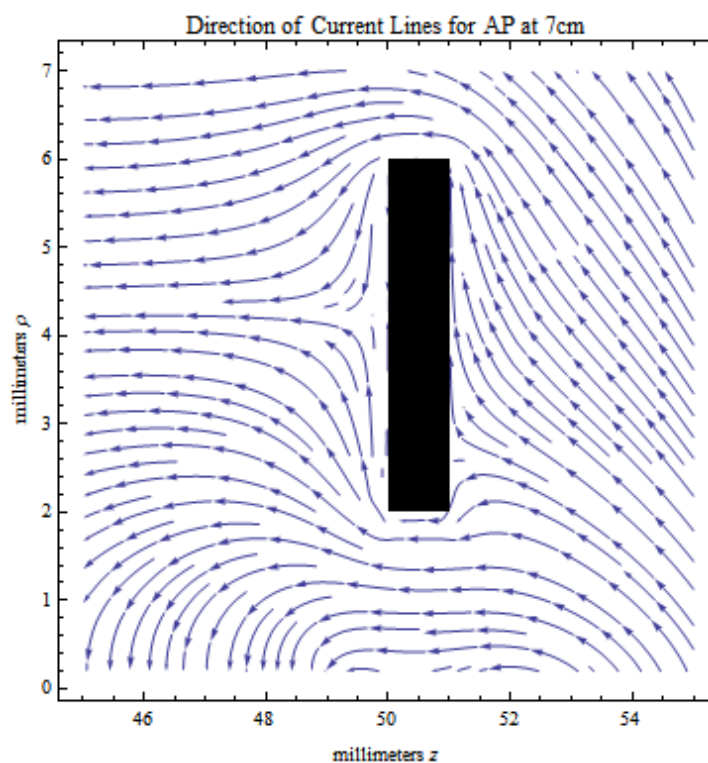


Figure 4.26 Current plot for  $r = 2$  mm

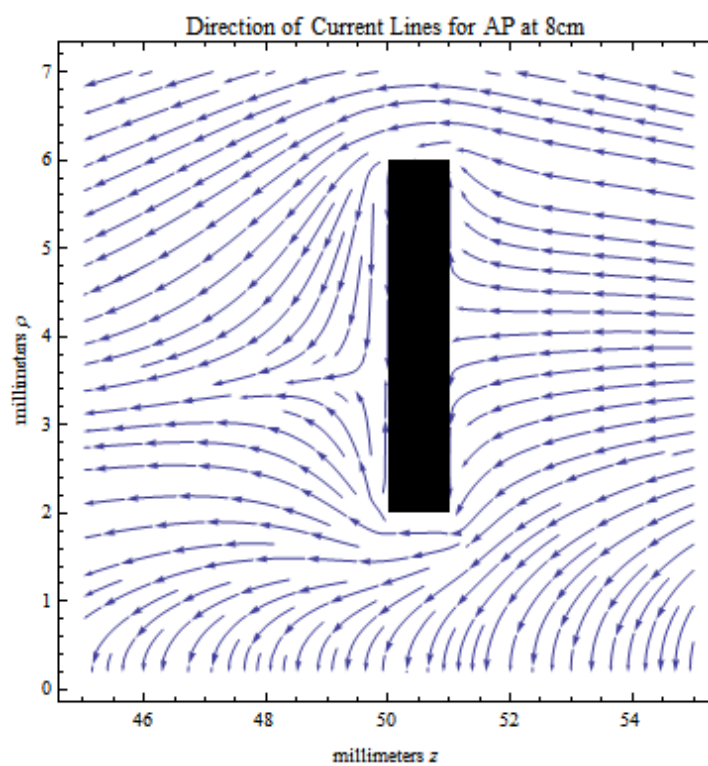


Figure 4.27 Current plot for  $r = 2$  mm

## CHAPTER 5

### Conclusion

#### 5.1 Discussion of Results

The current plots for  $r = 1$  mm and  $r = 2$  mm have shown the direction of the current lines in the extracellular region when a toroid is surrounding the axon. Without the toroid present in the conducting bath, the figures should look like Figure 1.1.

However, as written earlier, the toroid creates a discontinuity in the external area causing the current to redistribute. The current must still flow in closed loops, but now has to flow around the toroid. The plots have demonstrated just how the current will behave. After reviewing the current tables above, as the toroid radius increases, an increasing portion of the external current returns back through the toroid. The return current will decrease the net current within the closed path of integration when using Ampere's law to calculate the external magnetic field. Therefore, the measured magnetic field is dependent upon the toroid radius. It is difficult for a toroid to fit around a nerve perfectly, with no space between the two surfaces. The current tables have shown that even for a small toroid radius, there is still a return current that will affect the measured magnetic field. It is true, however, that a smaller toroid radius will have less of an effect

than a larger toroid radius. The current tables go further and also show that there is more of an impact on the net current when the AP is near the toroid. The largest values for the return current were seen when the AP was 4-6 cm along the nerve. Although I have only changed one parameter for each of my simulations, the mathematical model has been set up to easily change other parameters. The basic size of the plastic container that holds the axon, the toroid dimensions and positioning, as well as, the number of toroids surrounding the axon can all be changed accordingly to the experimental setup. Also, each different axon used has a different length, radius, and conduction velocity and those values can all be changed too.

## **5.2 Short – Comings of the Model**

The main issue arises from the interior and surface potential values that are read into the program at the start (Figures 4.1 and 4.2). It is these values that ultimately determine the extracellular and intracellular current values. The files I used were both 256 points long, whereas the axon is represented by 501 points. Therefore, I have had to fill in the extra points with zeros either before or after the AP depending on its location along the axon. In actuality, the values may be very small, but not zero. The effects are evident in both the current line plots, as well as, the current value tables. There should be no place in the extracellular region that has zero current. However, this is seen in Figures 4.16 – 4.19. There, also, should not be zero interior current at any snapshot along the nerve. A possible solution to this problem, may be to use experimentally gathered data for the surface potential of the axon and depending on its size, acquire interior potential data, as well. One may also have noticed that for  $r = 0.4$  mm, there is zero return current

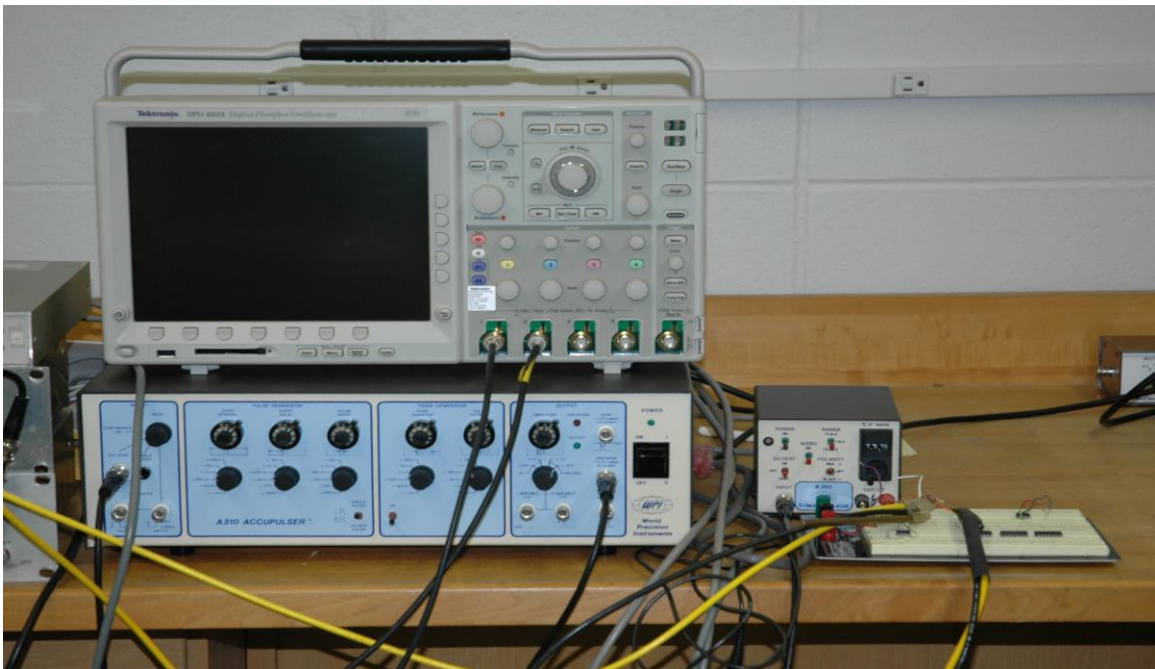
when the AP is at 1cm, 2cm, and 8cm. The return current should be small, but not zero. This is caused by both the setup of the mathematical model and the inserting of zeros into the surface potential signal. In order to satisfy Neumann's Condition at the boundary of the toroid (no radial current), the potential value on the surface of the toroid is set equal to the adjacent grid potential value. However, for  $r = 0.4$  mm, the adjacent grid potential value is the potential on the surface of the axon, because the axon radius is 2 mm  $\Delta p = 2$  mm. As a result, if the value of the surface potential is zero, so is the corresponding value on the surface of the toroid. Therefore, the return current will be calculated as zero, which is incorrect. This problem could also be solved by having a better estimation of the surface action potential. Referring back to the current plots, notice that near the boundaries of the plastic container, the current lines do not appear to flow in continuous loops. There should be no current flowing into or out of the outside walls. As these are all issues to address, keep in mind the goal of the paper is to calculate the return current and its effect on the magnetic field. I think the mathematical program does this task.

### **5.3 Recommendations for Further Research**

Experimental data must be taken, in order to validate the theoretical model presented in this paper. In order to make a comparison, systematic errors in the data must be taken into account. A bullfrog's sciatic nerve will be dissected and placed in a purified Ringer's solution and threaded through the center of toroids with different radii. Ball State has the proper equipment to measure the magnetic field associated with a stimulated nerve. The lab equipment is shown below and is set up to record data.



**Figure 5.1** The two smaller boxes shown are the low noise, room temperature current – voltage amplifiers. The two larger boxes shown are the control boxes which provide frequency compensation, calibration pulses, and triggering pulses.



**Figure 5.2** An oscilloscope which can act as a signal averager, sits on top of a pulse generator that triggers the small current stimulator to the right.

I suggest rewriting the program in MatLab or some other computer program designed to run very large matrices. I had to have step sizes for  $z$  and  $\rho$  no smaller than 2 mm, because otherwise the grid network was too large and the computation times were long. Therefore, there was a restriction on how small I could make the dimensions of the toroid and the toroid radius. By decreasing the step size, the calculated values would be more accurate. Another option would be to create a larger matrix, but run the Mathematica program on the Cluster.

Hopefully, this work will be another step forward in better determining the extent of nerve damage in humans. A clip-on current probe has been designed and has the ability to measure the currents flowing in a nerve and thus the continuity of axons at the site of an injury [16]. This is a non-invasive approach in testing for nerve damage. Using the recording techniques outlined in the experimental setup, may significantly simplify intraoperative nerve recordings. The procedure is safer, because the nerve remains within its normal physiological internal environment during recording. Other techniques require a pair of electrodes to suspend the nerve in air and require careful electrode placement and dissection of nerve. It is very important to keep the portion of the nerve between the electrodes dry during stimulation, while assuring that it does not dry out to the point of cell death. Biomagnetic recordings of nerve action potentials may offer a better solution for measuring nerve action currents [7].



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## APPENDIX A

### AP1.nb (Return Current for AP at 1cm)

Read in Initial Potential Grid (from Excel) with AP starting at 1cm along the nerve. Use the SOR method to relax potential over entire grid. Toroid radius = 1mm.

```

Clear[L]
Z=Table[i,{i,0,100,0.2}];
P = Table[j,{j,0.2,25,0.2}];
L=Import["E:\\AP3-1cm.xls"][[1]];
L=Drop[L,1];
M=100;
w=1.75;
TTLR=251; TTLC=5; TTRR= 251; TTRC= 25; TBRR=256; TBRC=25; TBLR=256;
TBLC = 5;
TableForm[L,TableHeadings->{Z,P}];
A=Table[{Z[[i]],P[[j]],L[[i,j]]},{i,1,Length[Z]},{j,1,Length[P]}];
Print["Initial External Potential at 1cm. (Before Relaxation)"]
ListPlot3D[{A},PlotRange->All,AxesLabel->{z (mm),p (mm),μV}];
Print["****"]For[k=1,k<=M,k++,
  For[i=3,i<=Length[Z]-2,i++,For[j=2,j<=Length[P]-2,j++,If[TTLR<= i<= TBLR
&& TTLC <= j <= TTRC,L[[i,j]]=L[[i,j]],L[[i,j]]=
  (1-w)*L[[i,j]]+(w/4)*(L[[i,j+1]]*(1+(0.2/(2*P[[j]])))+L[[i,j-
1]]*(1-(0.2/(2*P[[j]])))+1*(L[[i+1,j]]+L[[i-1,j]]))]];
  For[i=TTLR,i<= TBLR,i++, For[j=TTLC,j<= TTRC,j++,
If[i<=TTLR&&(TTLC+1)<=j<= (TTRC-1),L[[i,j]]=L[[i-1,j]],L[[i,j]]];If[TTRR<=
i<= TBRR&&j<=TTRC,L[[i,j]]=L[[i,j+1]],L[[i,j]]];
  If[i<=TBLR&&(TTLC+1)<= j<= (TTRC-1),L[[i,j]]=L[[i+1,j]],L[[i,j]]];
If[TTLR<= i<=TBLR&&j<=TTLC, L[[i,j]]=L[[i,j-1]],L[[i,j]]
  ]]];
Print[k-1]
TableForm[L,TableHeadings->{Z,P}];
B=Table[{Z[[i]],P[[j]],L[[i,j]]},{i,1,Length[Z]},{j,1,Length[P]}];
Print["Relaxed External Potential at 1cm"]
ListPlot3D[{B},PlotRange-> All,AxesLabel->{z (mm),p (mm),mV}]
Relax1=Table[L[[i,j]],{i,1,Length[Z]},{j,1,Length[P]}];
TableForm[Relax1,TableHeadings->{Z,P}];
VC=Table[L[[i,j]],{i,1,Length[Z]},{j,1,Length[P]}];
TableForm[VC,TableHeadings->{Z,P}];
HC=Table[L[[i,j]],{i,1,Length[Z]},{j,1,Length[P]}];

```

```
TableForm[HC, TableHeadings → {Z, P}];
```

Determine the extracellular resistance per unit length.

dz is in mm,  $\sigma_e$  is in  $\text{ohms}^{-1} \cdot \text{m}^{-1}$  (Wije article)

Resistance will be in KOhms.

```
dz=0.2;  $\sigma_e$ =1.20;
```

```
R = 1/ $\sigma_e$ *dz)
```

#### CURRENT CALCULATIONS USING RELAXED POTENTIAL GRID VALUES

Replace empty entries inside toroid with zeros. There is zero current inside toroid.

```
For[i=1, i ≤ Length[Z], i++, For[j=1, j ≤ Length[P], j++, If[TTLR+1 ≤ i ≤ TBLR-1 && TTLC+1 ≤ j ≤ TTRC-1, HC[[i, j]] = 0, HC[[i, j]] = HC[[i, j]]]]];
```

Calculate horizontal current (z component) at each point in extracellular potential grid. Use  $I = dV/R$ .

With Voltage in microVolts and R in KOhms, the current is in nanoamps.

```
For[i=1, i ≤ Length[Z]-1, i++, For[j=1, j ≤ Length[P], j++, HC[[i, j]] = (HC[[i+1, j]] - HC[[i, j]])/R];
For[i=TTLR, i ≤ TBLR-1, i++, For[j=TTLC+1, j ≤ TTRC-1, j++, HC[[i, j]] = 0];
For[j=1, j ≤ Length[P], j++, HC[[Length[Z], j]] = 0];
HC//TableForm;
```

```
For[i=1, i ≤ Length[Z], i++, For[j=1, j ≤ Length[P], j++, If[TTLR+1 ≤ i ≤ TBLR-1 && TTLC+1 ≤ j ≤ TTRC-1, VC[[i, j]] = 0, VC[[i, j]] = VC[[i, j]]]]];
```

Calculate vertical current ( $\rho$  component) at each point in extracellular potential grid. Use  $I = dV/R$ . (nanoamps)

```
For[i=1, i ≤ Length[Z], i++, For[j=1, j ≤ Length[P]-1, j++, VC[[i, j]] = (VC[[i, j+1]] - VC[[i, j]])/R];
For[i=TTLR+1, i ≤ TBLR-1, i++, For[j=TTLC, j ≤ TTRC-1, j++, VC[[i, j]] = 0];
For[i=1, i ≤ Length[Z], i++, VC[[i, Length[P]]] = 0];
VC//TableForm;
```

Vector Plot of current at 1cm; shown for entire length of nerve (100mm) and conducting bath (25mm).

Toroid is positioned at  $z = 50\text{mm}$  and extends to  $z = 51\text{mm}$  and  $p = 1\text{mm}$  and extends to  $p = 5\text{mm}$ .

```
TBLCZ=50; TBLCP=1; TTRCZ=51; TTRCP=5;
```

```
MagCurrent= Table[{Z[[i]], P[[j]]}, {-HC[[i, j]], -VC[[i, j]]}, {i, 1, Length[Z]}, {j, 1, Length[P]}];
Current=ListStreamPlot[MagCurrent, FrameLabel → {z millimeters,  $\rho$  millimeters}, PerformanceGoal → "Quality", StreamPoints → Fine, PlotLabel → "Direction of Current Lines for AP at 1cm"];
Toroid=Graphics[{Rectangle[{TBLCZ+.1, TBLCP+.1}, {TTRCZ-.1, TTRCP-.1}]}];
Show[{Current, Toroid}]
```

Zoom in near toroid. Change scale.

```

MagCurrent= Table[{Z[[i]],P[[j]]},{-HC[[i,j]],-
VC[[i,j]]},{i,226,276},{j,1,30}];
Current=ListStreamPlot[MagCurrent,FrameLabel->{z (millimeters),ρ
(millimeters)},PerformanceGoal->"Quality",StreamColorFunction->Red,Stream
mPoints->Fine,PlotLabel->"Direction of Current Lines for AP at 1cm" ];
Toroid=Graphics[{Rectangle[{TBL CZ,TBL CP},{TTR CZ,TTR CP}]}];
Show[{Current,Toroid}]

```

Calculate Total Return Current through toroid at  $z = 50\text{mm}$ . Toroid starts at  $z = 50\text{mm}$  or  $Z[[\text{TTLR}]]$ ,  $\rho = 1$  or  $P[[\text{TTL C}]]$ . (using only Horizontal Component of current, HC)

RC = return current in nanoamps

```

RC=0;
For[j =1, j≤ TTL C-1,j++,RC1=RC+( (P[[j+1]]^2*π-P[[j]]^2*π)/(dz)^2)*(-
HC[[TTL R,j]]*(dz)^2);
RC=RC1;
Print["RC = " ]Print[RC]]

```

Calculate net current through toroid by finding the primary current through the toroid (intracellular current in axon) and adding the total return current (extracellular current).

Input Potential Values from Exel File.

```
PV=Partition[Flatten[ Import["E:\\InteriorPotential3-1cm.xls"]],1];
```

$r$  is axial resistance per unit length;  $dz$  is mesh space size;  $\sigma_i$  is Intracellular conductivity.  
 $dz$  is in mm,  $\sigma_i$  is in  $\text{ohms}^{-1} \cdot \text{m}^{-1}$   
 Resistance will be in KOhms.

```
dz=0.2;σi=0.88;r=1/(σi*dz)
```

Calculate intracellular current (IC) using  $I = (-dV/dz)/r$ .

```

IC=Table[PV[[i]],{i,1,Length[Z]}];
For[i=1,i≤ Length[Z]-1,i++,IC[[i]]=(IC[[i+1]]-IC[[i]])/(dz*r)];
IC[[Length[Z]]]=0;

```

Determine Intracellular Current at  $z$  - value corresponding to start of toroid.

```
IC[[TTL R]]
```

Extracellular current ( $I = dV/R$ ) is microvolts/KOhms, which gives nanoamps. Intracellular current ( $I = -dV/(dz*r)$ ) is mVolts divided by mm \* KOhms, which gives microA per mm.

Last step is to put the return current and interior current in the same units (microamps) and add the two values at  $z = 50\text{mm}$  to get the net current (NC) through the toroid.

```

NC = IC[[TTL R]]+(RC*10^-3);
Print["Net Current through Toroid in microamps = " ]Print[NC]

```

